Formulas and Facts: A Response to John Gee

Andrew W. Cook[1]

The Story Continues

In winter 2010, Chris Smith and I published an article in *Dialogue* demonstrating that no more than ~56 cm of papyrus can be missing from the interior of the scroll of Hôr—the papyrus Joseph Smith identified as the Book of Abraham.[2] John Gee has responded by claiming that our method is "anything but accurate" and that it "glaringly underestimates the length of the scroll."[3] He states that "Two different formulas have been published for estimating the original length of a scroll," then attempts to show that "Hoffmann's formula approximates the actual length of the papyrus," whereas "Cook and Smith's formula predicts a highly inaccurate length." The fact is, the two formulas are completely equivalent. They are both exact expressions of an Archimedean spiral and they yield precisely the same results, if correctly applied.

A Tragedy of Errors

Gee has confused differences in *notation* and *convention* with differences in the formula itself. Hoffmann's *expression* for the spiral formula is: $Z=(E^2-6.25)/(2S)-E+S/2$, where Z is the length of the missing interior section of a spiral, E is the length of the innermost extant winding and S is the average difference in length between successive windings.[4] We *expressed* the spiral formula as: $L=(W^2-2.5^2)/(4\pi T)$, where $T=S/(2\pi)$, $W=E$ and $L=Z+E-S/2$. (Our centered convention for the winding numbers and definition of where the missing section begins removed the factor of $-E+S/2$ from the right-hand side.) In other words, Hoffmann's Z, E and S variables are freely interchangeable with Cook/Smith's L, W and T variables, using the relations: $Z=L-E+S/2$, $E=W$ and $S=2\pi T$.

Plugging these relations into Hoffmann's equation converts it to the Cook/Smith format. Likewise, the Cook/Smith equation is readily transformed into Hoffmann's format by straightforward substitution. Properly applied, the "Hoffmann formula" and the "Cook/Smith formula" give *identical* predictions for the missing length because they are mathematically equivalent.

How then, did Gee manage to obtain such wildly different results from the two equivalent expressions for the same spiral formula? It's hard to say for certain, since he doesn't report any winding measurements or other basic information necessary to check his work. However, his comments and results strongly suggest that, in applying the "Cook/Smith Formula," he used the wrong T parameter appearing in the denominator of equation (3) in our 2010 paper. We called this parameter "effective thickness," since it represents the average increase in *radius* of the (wound up) scroll with each 360 degree wrap of papyrus. It plays the same role as Hoffmann's S factor, which represents the average increase in *circumference* of the scroll with each 360 degree wrap. When the scroll is unrolled, circumference becomes *winding length*. Just as the radius of a circle can be computed from its circumference, so too can effective thickness (T) be computed from winding length (S). The T parameter derives from winding lengths and equation (4) in our paper is another way of saying $T=S/(2\pi)$. It appears that Gee has ignored this essential fact, since he describes his methodology as follows, "I applied each of the mathematical formulas, using the assumptions made by the authors of the formulas concerning papyrus thickness, air-gap size, and size of smallest interior winding." Neither papyrus thickness nor air-gap size has anything to do with the equations in our paper and we made no assumptions concerning them. As discussed below, it seems that Gee has erroneously applied the T value we reported for the Hôr scroll to ROM 910.85.236.1-.13, a 332–330 BC Book of the Dead for a man named Amenemhet.[5]

Some Puzzles from the John Gee Paper

The only quantitative result in Gee's paper is a plot of the length of Papyrus ROM 910.85.236.1-.13 vs. winding number. It contains a blue curve, a purple curve and a green curve. The green curve is labeled "Cook/Smith Formula" as though it had something to do with the formula in our paper. The purple and blue curves, respectively labeled "Hoffmann Formula" and "Actual Length," lie well above the green curve. Obviously, if the green (Cook/Smith) and purple (Hoffmann) curves had been correctly plotted, they should have lain directly on top of each other. The only way to generate a difference between the green and purple curves is to feed them different inputs; e.g., set $T > S/(2\pi)$ for the green curve, where S was used for the purple curve. In an effort to justify the altered inputs for the green curve, Gee wrongly declares that "Cook and Smith use the thickness of the papyri (which they did not measure but only estimated) as an indication of the change in diameter to calculate the difference between the lengths of successive windings in the scroll." On the contrary, we did not use or estimate the material thickness of the papyri in any manner in our calculations. We plainly stated that physical thickness cannot be used to estimate missing length due to the many additional unknowns involved, such as Gee's "air-gap size." Gee has stated our method exactly backwards; we did not use thickness to calculate winding differences, rather we used the winding differences to calculate T (essentially unrelated to the *physical thickness,* except that it must be greater). The T factor is purely a *derived parameter of convenience*; i.e., T can be *entirely removed* from the spiral formula by simply combining equations (3) and (4) in our paper. The spiral formula (be it Hoffmann's expression or Cook/Smith's expression) should receive winding lengths as inputs and nothing else.

Undeterred by the actual content of our paper, Gee proceeds to contrast the blundering Cook/Smith with the wise and steady Hoffmann; "Hoffmann—knowing that most papyri are already mounted, thus rendering it impossible to measure the thickness—uses the average difference between successive windings for the same purpose." Had Gee made a genuine effort to understand our methodology, he might have realized that we applied the very same "average difference" technique as Hoffmann; i.e., we derived the effective thickness (expressed as T or S) from the windings (W), not the windings from the thickness, as he alleges. We explicitly stated, "Our primary task therefore, is to determine the effective thickness of the papyrus from the winding lengths."

And we expressed this statement mathematically in equation (4) of our paper, which Gee disregarded.

The green (Cook/Smith) curve is not only shifted downward with respect to the other curves but it is also much smoother than the purple (Hoffmann) curve. This may be a result of Gee using multiple (local) values of S in the "Hoffmann Formula" (combined with inaccurate winding measurements) but only a single (global) value of S (or T) in the "Cook/Smith Formula." Consistency would, of course, have required that either the local or global method be used for both formulas. However, if the green curve had received the same "erratic" inputs as the purple curve then it would have occasionally crossed the blue (Actual Length) line. This might have given some readers the impression that the "Cook/Smith formula" could occasionally produce the right answer. It appears that Gee could not tolerate such an outcome, since the green curve exhibits a systematic shift in both the *magnitude* and *variance* of the input data, thus keeping it comfortably below the blue curve for all winding numbers. To bolster this satisfying result, Gee's editor assures us that "John Gee has tackled this relative question with objectivity and precision."

New Light on the Amenemhet Papyrus

With the gracious assistance of Janet Cowan, the ROM's paper conservator, and Irmtraut Munro, an Egyptologist at the University of Bonn, I obtained a complete set of winding measurements for Papyrus ROM 910.85.236.1-.13.[6] After performing basic consistency checks and cross validations against the measurements of Cowan and Munro, I evaluated Gee's calculations by applying each version of the spiral formula to the first 73 (contiguous) windings of the scroll. The 1st (innermost) winding measures 3.40 cm and the 73rd winding measures 11.30 cm; hence, the S factor for this scroll is (11.30–3.40)/(73-1)=0.11 cm (T=0.0175 cm).[7] Using this S factor, I plugged each winding length into the "Hoffmann Formula" and the "Cook/Smith Formula" and computed the length of the scroll at each winding number. (This appears to be what Gee did in evaluating the "Cook/Smith Formula," except that here I've used the correct S factor.) The results are seen in Figure 2, which should be compared to the plot in Gee's paper.

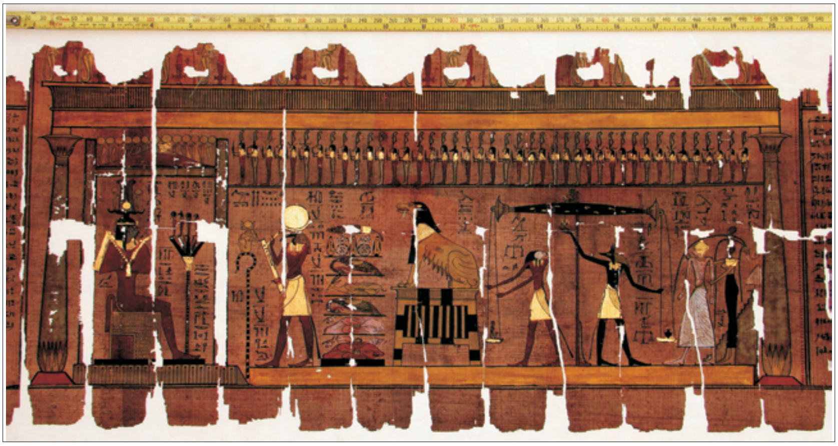

Figure 1. *Windings 46–51 (papyrus section 910.85.236.10) of the Amenemhet Book of the Dead. As in Facsimile 3 in the Book of Abraham, the deceased is accompanied by Maat and Anubis into the Hall of Two Truths where his deeds are judged before the throne of Osiris. Courtesy of the Royal Ontario Museum, © ROM.*

Given the same inputs, the two versions of the spiral formula predict exactly the same papyrus length, regardless of location (winding number). Properly applied, the spiral formula gives excellent predictions for the length of this scroll because the windings exhibit a nearly *linear* progression; i.e., they increase by an almost constant amount from one winding to the next.[8] Archimedean spirals possess this very same property; in fact, a linear winding progression *defines* an Archimedean spiral. Hoffmann provides a nice example of linear winding progression in Figure 3 of his paper, wherein he plots the windings of Papyrus Spiegelberg as vertical bars and draws a straight line through their end points. The slope of Hoffmann's line sets the S factor (average change in length between windings) for P. Spiegelberg to 0.44 cm.

For the Hôr scroll, we also reported an S factor of 0.44 cm.[9] This is four times larger than the S factor of the Amenemhet scroll, which further indicates that, in evaluating the "Cook/ Smith Formula," Gee misapplied the Hôr scroll's S factor to the

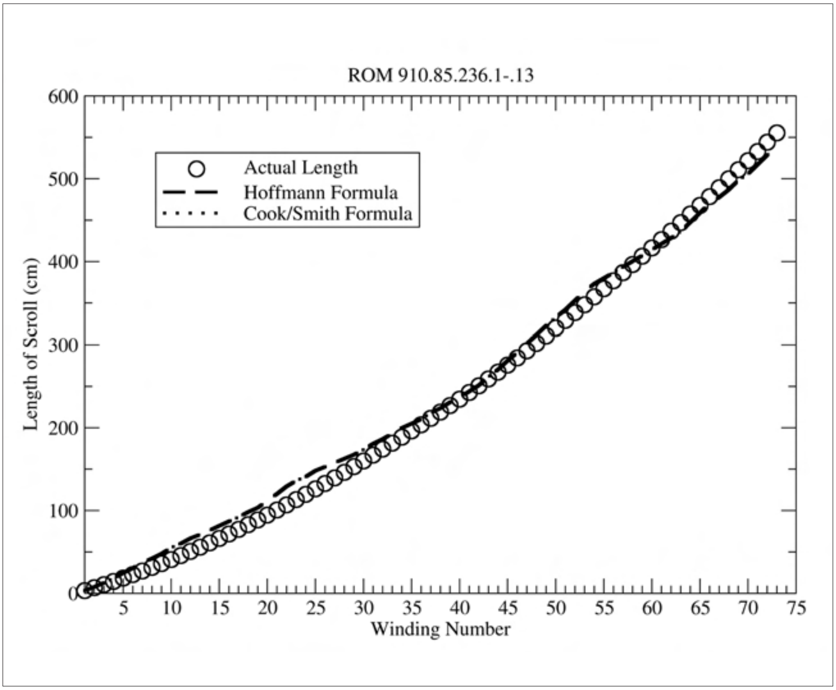

Figure 2. *Comparison of actual scroll length to predicted scroll length using the two versions of the spiral formula. The dashed and dotted lines lie on top of one another. The oscillations in the dashed and dotted lines are due to uncertainty in the winding measurements.*

Amenemhet scroll. Gee notes that "Cook and Smith's formula also improves with more data, ranging from about a quarter of the correct length to about a third of the correct length." The factor-of-four difference in S between the two scrolls appears to account for Gee's "quarter of the correct length" at larger winding numbers. Furthermore, if S is computed locally for the Amenemhet papyrus, it increases to about 0.147 cm at the core of the scroll, or about a third of the Hôr scroll's S factor. This would account for Gee's "third of the correct length" at lower winding numbers. Interestingly, had Gee been consistent (albeit wrong) and applied P. Spiegelberg's S factor to the Amenemhet scroll, as his input for the Hoffmann formula, his purple and green curves would have overlain each other.

Figure 3. *Top half of papyrus ROM 910.85.236.1-.13 as it appeared during the unrolling process. Courtesy of the Royal Ontario Museum, ©ROM.*

Just the Facts

Gee's attempt to cast doubt on the spiral formula is a red herring. The formula is exact for Archimedean spirals and such spirals are excellent models of papyrus scrolls.[10] We needn't fear that there may be "some errors in it or in the assumptions upon which it is based." Fundamentally, a scroll's length is simply the sum of its windings. Another way of determining a scroll's original length, which involves less math, is to plot the lengths of the extant windings and fit a straight line to the results. The missing windings will reliably lie along the straight line. The spiral formula is just a convenient way of adding up all the missing windings. What really matters is that the extant windings be accurately measured.

The heart of our 2010 paper was not the spiral formula but rather the autocorrelation method for more accurate and reliable determination of the winding lengths. The method returns non-integer winding numbers, from which we derived the T parameter for the extant sections of the Hôr scroll. To simplify our results and facilitate comparisons, I have converted our winding numbers to integers by using the T value we found from the autocorrelation analysis. Numbering from the beginning (outside) of the papyrus inward (right to left), the extant windings of the Hôr scroll are (in centimeters): W_1=10.64, W_2=10.21, W_3=9.77, W_4=[9.32], W_5=8.86, W_6=8.39 & W_7=7.91.[11] Continuing this progression for the missing windings yields: W_8=[7.43], W_9=[6.95], W_{10}=[6.47], W_{11}=[5.99], W_{12}=[5.51], W_{13}=[5.03], W_{14}=[4.55], W_{15}=[4.07], W_{16}=[3.59], W_{17}=[3.11], W_{18}=[2.63] & W_{19}=[2.15]. The length of missing papyrus can be determined by manually adding up these numbers. This simpler procedure requires neither formulas nor faith, only "objectivity and precision."

Notes

1. I am grateful to Chris Smith for his valuable insights and helpful comments on the various drafts of this paper.

2. Andrew W. Cook and Christopher C. Smith, "The Original Length of the Scroll of Hôr," *Dialogue: AMormon Thought* 43, no. 4 (Winter 2010): 1–42. For a comprehensive treatment of all the Joseph Smith Papyri, see Christopher C. Smith, "That Which Is Lost: Assessing the State of Preservation of the Joseph Smith Papyri," *The John Whitmer His-*

torical Association Journal 31, no. 1 (Spring/Summer 2011): 69–83. For a full translation of the Joseph Smith Papyri, see Robert K. Ritner, *The Joseph Smith Egyptian Papyri: A Complete Edition*, (Salt Lake City: Smith-Pettit Foundation, 2012).

3. John Gee, "Formulas and Faith," *Journal of the Book of Mormon and Other Restoration Scripture* 21, no. 1 (2012): 60–65. See also John Gee, "Book of Abraham, I Presume," presentation delivered at the FAIR conference on Aug. 3, 2012, http://www.fairlds.org/fair-conferences/2012-fair-conference/2012-book-of-abraham-i-presume.

4. Friedhelm Hoffmann, "Die Länge Des P. Spiegelberg," in *Acta Demotica: Acts of Fifth International Conference for Demotists* (Pisa, Italy: Giardini Editori e Stampatori, 1994), 145–155.

5. Gee refers to this papyrus as ROM 978x43.1; however, the Royal Ontario Museum no longer considers this accession number to be correct. It was assigned in 1978 but the museum has since found the original number to be 910.85.236.1-.13. The Museum has requested that this original number be used in correspondence and publications referring to this scroll.

6. These data are available for download from the *Dialogue* website.

7. This is an unusually small value for Ptolemaic papyrus. When I presented these results to Irmtraut Munro, she replied, "Indeed the papyrus was the thinnest material I have ever seen, so that in some cases two sheets stuck together."

8. The slight over prediction at small winding numbers is due to the fact that the inner windings are a little looser than the outer windings, as determined by direct measurements.

9. A recent correction suggests the S factor for the Hôr scroll should be closer to 0.48 cm. Page 29 of our *Dialogue* (2010) paper contains an error, which unfortunately carried through some of the arithmetic. The "2.221" should be "1.665" leading to T=0.0859 cm, rather than T=0.0649 cm. Averaging the three reliable estimates yields T=0.0771 cm, rather than T=0.0701 cm. This changes the estimate of the missing papyrus length from 56 cm to 51 cm.

10. In Hartmut Stegemann's study of the Dead Sea Scrolls, he found that, ". . .if the material involved is 0.8 mm papyrus [this refers to effective thickness (T) not physical thickness], the increase or decrease [from one winding to the next] is always about 5 mm. One can measure this arithmetic progression with exactitude in all of the larger Qumran scrolls." Hartmut Stegemann, "Methods for the Reconstruction of Scrolls from Scattered Fragments," in *Archaeology and History in the Dead Sea Scrolls: The New York University Conference in Memory of Yigael Yadin*,

edited by Lawrence H. Schiffman, JSOT/ASOR MONOGRAPH SERIES (Sheffield, England: JSOT Press, 1990), 194–197.

11. Some of winding 4 falls in the gap between pJS 1.2 and 1.3; nevertheless, its length can be interpolated along with the other windings. Based on the scatter in T, each of these winding lengths should be accurate to plus or minus half a millimeter.