The Original Length of the Scroll of Hôr

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These records were torn by being taken from the roll of embalming salve which contained them, and some parts entirely lost; but Smith is to translate the whole by divine inspiration, and that which is lost, like Nebuchadnezzar’s dream, can be interpreted as well as that which is preserved; and a larger volume than the Bible will be required to contain them.—William S. West (1837)

/Ed. note—Figures 1–4 and 25–28 contain features not visible in black and white. For the tracings etc., please see the color version of the article online at http://dialoguejournal.com/2010/the-original-length-of-the-scroll-of-hor./

The Story So Far

Early in the second century B.C., an Egyptian scribe copied a Document of Breathing Made by Isis onto a roll of papyrus for a Theban priest named Hôr. Near the beginning of the document, the scribe penned the following set of ritual instructions: “The Breathing Document, being what is written on its interior and exterior, shall be wrapped in royal linen and placed (under) his left arm in the midst of his heart. The remainder of his wrapping shall be made over it.” Hôr’s mummy, with the Breathing Document enclosed, was buried in a pit tomb near Thebes, where it lay undisturbed for two millennia.

Sometime around 1820, Italian adventurer Antonio Lebolo exhumed a cache of mummies, including Hôr. After Lebolo’s death in February 1830, eleven of his mummies were sold to benefit his children. The mummies were shipped to New York and then forwarded to maritime merchants in Philadelphia, where they were examined by medical doctors and exhibited in the Philadelphia Arcade. At some point, the mummies were delivered to a traveling showman named Michael H. Chandler for further exhibition. Chandler reportedly unwrapped them in search of valuables. On two of the bodies, he found papyrus scrolls wrapped in linen and saturated with a bitumen preservative. As he extracted
the Hôr scroll from its sticky encasement, the edges were torn, thus imprinting a repeating pattern of lacunae in the papyrus.

Chandler eventually made his way to Kirtland, Ohio, where he sold the Hôr scroll, along with four mummies, a Book of the Dead scroll made for a woman named Tshenmîn, a Book of the Dead fragment bearing the female name Neferirimûb, another fragment bearing the male name Amenhotep, and a hypocephalus belonging to a man named Sheshonk to Joseph Smith in July 1835 for $2,400. Shortly after the purchase, Smith claimed that one of the rolls in his possession contained a record of the biblical patriarch Abraham, which he began to translate by the gift and power of God. Although Smith died before he could finish the work, his partial translation of the Book of Abraham was canonized in 1880 as part of the Pearl of Great Price. In addition to five chapters of Jacobean English prose, the book includes facsimiles of three vignettes from the papyri: i.e., the hypocephalus of Sheshonk and the introductory and concluding vignettes of the Document of Breathing. The introductory vignette, labeled “Facsimile 1” in the canonized LDS Pearl of Great Price, is said in the text of the Book of Abraham to have appeared “at the commencement” and “at the beginning” of Abraham’s record (Abr. 1:12, 14). This and other evidence points to the Hôr scroll as the papyrus from which Joseph Smith claimed to translate the Book of Abraham.

Prior to Smith’s death, he or one of his associates glued the fragmented outer portion of the Document of Breathing onto stiff paper in an effort to preserve it. Some of the mounted fragments were then cut into shorter sections and preserved under glass. By mounting the outer sections, Smith et al. could work on translating the Egyptian characters without needing to roll and unroll the fragile scroll. After Smith was assassinated in 1844, the mummies and papyri were retained by his mother, Lucy Mack Smith, and briefly taken on an exhibition tour by Joseph’s only surviving brother, William. When Lucy died in 1856, Joseph’s widow, Emma, and her second husband, Lewis Bidamon, sold the artifacts to Abel Combs. Combs divided the collection into two parts. One part, including the intact interior portion of the Hôr scroll, he sold to Wyman’s Museum in St. Louis, which subsequently relocated to Chicago and burned in 1871. The other part,
including the mounted fragments from the outer portion of the Hôr scroll, he retained and eventually left to his housekeeper, whose daughter’s widower sold them in 1947 to the New York Metropolitan Museum of Art. The museum turned this portion of the collection over to the Church of Jesus Christ of Latter-day Saints on November 27, 1967.15

When the papyri were recovered by the Church, it was immediately evident that the Hôr scroll was the source of Facsimile 1. There are also several 1835 manuscripts in the handwriting of Joseph Smith’s known scribes that juxtapose the translated Book of Abraham text with sequential characters from the scroll’s extant instructions column, ostensibly as the source from which the translation was derived.16 Some LDS historians nevertheless maintain that the source from which Joseph Smith derived the Book of Abraham is not among the extant fragments, and that it was probably destroyed with that portion of the collection which burned in the Great Chicago Fire of 1871. These authors have argued that the Hôr scroll was much longer in the nineteenth century than it is today and that the source text of the Book of Abraham may have followed the Document of Breathing on the now-lost inner portion of the scroll. In the view of these researchers, the Book of Abraham’s placement of Facsimile 1 “at the commencement of this record” should be interpreted to mean the beginning of the scroll rather than the record, and the juxtaposition of Breathing Document characters with the Book of Abraham’s English text in the handwritten manuscripts should not be understood to imply a translation relationship between the two.17

The question then becomes whether the undamaged scroll of Hôr was ever long enough to accommodate a hieratic Book of Abraham source text. The main text of the canonized Book of Abraham contains 5,506 English words. The hieratic text in the instructions column of the Document of Breathing translates to ~97 English words.18 This column is ~9 cm wide. Hence, if the Book of Abraham was written on the scroll in the same hieratic font as this portion of the Document of Breathing, it would have taken up ~9(5,506/97) = ~511 cm of papyrus. Since the Book of Abraham translation is incomplete, the actual space required for a hieratic original would presumably have been even longer.19
Recently, John Gee proposed that 1250.5 cm (41 feet) of papyrus could be missing from the interior end of the scroll of Hôr. This is obviously more than enough papyrus to contain the extant Book of Abraham. Gee followed an approach pioneered by Friedrich Hoffmann, which takes advantage of the fact that “the circumference of a scroll limits the amount of scroll that can be contained inside it. Thus, we can determine by the size of the circumference and the tightness of the winding how much papyrus can be missing at the interior end of a papyrus roll.”\textsuperscript{20} Gee reported 9.7 and 9.5 cm as the lengths of the first and seventh windings, re-
spectively, but offered no details concerning his method for identifying the winding end-points. When we attempted to replicate Gee’s results, we found that his measurements did not seem to be accurate and, in fact, required the papyrus to be impossibly thin.

The purpose of this paper is to introduce a robust methodology that eliminates the guesswork in determining winding locations by visual inspection of crease marks or lacunae features, and to determine whether the missing interior section of the Hôr scroll could have been long enough to accommodate the Book of Abraham. Fortunately, this is a question that can be definitively answered by examining the physical characteristics of the extant portions of the scroll. The haste and greed of Michael Chandler provide the key to unlocking this mystery.

**Spiral Integration**

A roll of papyrus, viewed from either end, can be approximated by an Archimedean spiral. Such a spiral is illustrated in Figure 1, where the outermost (first) winding is colored blue and the next-to-outermost (second) winding is colored red. (See color illustration, www.dialoguejournal.com.) In an Archimedean spiral, the length and radius of each winding (proceeding inward) decreases by a constant amount per revolution. Note that the first (blue) winding is slightly longer than the second (red) winding and that there are twelve windings in total. We could compute the length of each black winding if we knew the lengths of the blue and red windings. Equivalently, we could compute the radius of each black winding if we knew the radii of the blue and red windings, since the distances across the white gaps (differences in radii between successive windings) are all the same.

Figure 1 is analogous to the Hôr scroll, the interior portion of which is missing (black windings), but the outer portion of which is extant (red and blue windings). The problem at hand involves significant complications to this simple example; nevertheless, our essential task is to determine the lengths of the extant outer windings of the scroll. Once these outer winding lengths are known, they can be fed into the formulas derived below to predict the total length of the missing interior windings. Although our formulas are derived specifically for an Archimedean spiral, the resulting model is valid for almost any topologi-
cally equivalent spiral, since it is hypothetically possible to distort the spiral in various ways, even bend it in half, without changing the winding lengths.

The length of a papyrus scroll can be computed (among other ways) via a path integral along the spiral. To accomplish this, we define the following variables:

\( \theta \) is the spiral angle, starting from the outside edge and proceeding inward. (Proceeding counterclockwise in the Figure 1 example, \( \theta \) begins at 0 on the outside edge and reaches a value of \( 2\pi \) at the inner end of the spiral.)

\( n = \theta / (2\pi) \) is the winding number; i.e., the number of revolutions around the spiral. (In Figure 1, \( n = 0 \) at the outside edge, \( n = 1 \) at the junction between the blue and red windings and \( n = 2 \) where the blue winding meets the first black winding.)

\( N \) is the total number of windings or revolutions from beginning to end. (In Figure 1, \( N = 12 \).)

\( W_n = 2\pi r_n \) is the length of the \( n \)th winding; i.e., the distance along the spiral from location \( (n - 1/2) \) to \( (n + 1/2) \). This centered definition avoids the messy extra terms appearing in Hoffmann’s derivation. (In Figure 1, \( W_{0.5} \) is the length of the blue winding and \( W_{1.5} \) is the length of the red winding.)

\( W_N \) is the winding length at the innermost end of the scroll. According to Hoffmann, “The windings cannot be put into practice under 2.5 cm”; hence we require \( W_N \geq 2.5 \text{ cm} \). \(^{21}\)

\( r_n = W_N / (2\pi) + (N - n)T \) is the radius of the angular center of \( W_n \). The angular center corresponds to the point halfway around the winding. (In Figure 1, the angular center of each winding corresponds to its leftmost point. Note that this is not the same as half the distance around each winding, since the radius is continually decreasing.)

\( T = (r_m - r_n) / (m - n) \) is the mean effective thickness of the papyrus between winding locations \( n \) and \( m \), accounting for wrinkling, inhomogeneities, eccentricity, etc. \(^{22}\) (\( T \) is the change of radius per winding, represented in Figure 1 by the white gaps between windings.)
The length of papyrus interior to any \( n \) location is (using \( m \) as a dummy index)

\[
L_n = \int_{2m}^{2N} r_m \, d\theta = \int_{2m}^{2N} \left[ \frac{W_N^2}{2\pi} + \left( N - \frac{\theta}{2\pi} \right) T \right] \, d\theta
\]

\[
= \pi T (N - n)^2 + W_N (N - n)
\]  \hspace{1cm} (1)

The number of windings between locations \( n \) and \( N \) depends on the winding lengths and effective thickness according to

\[
N - n = \frac{(W_n - W_N)}{2\pi T}
\]  \hspace{1cm} (2)

Combining equations (1) and (2) yields

\[
L_n = \frac{W_n^2 - W_N^2}{4\pi T}
\]  \hspace{1cm} (3)

Our primary task therefore, is to determine the effective thickness of the papyrus from the winding lengths; i.e.,

\[
T = \frac{W_n - W_m}{2\pi (m - n)}
\]  \hspace{1cm} (4)

In the following sections, we will describe our method for measuring winding lengths. It is based on a correlation analysis, which eliminates the inherent guesswork of visual observation techniques. Our investigation is somewhat more complicated than the Figure 1 example due to the fact that the increments in winding numbers (obtained from autocorrelation functions of the edges of the extant papyrus fragments) do not correspond to simple integers; i.e., the measured windings are unevenly spaced. Furthermore, we must work around a large gap of unknown width between two of the fragments. The gap width could be estimated from a textual analysis, but this is not precise enough for our purposes. The problem is well posed and the numbers are readily computable; however, in certain parts of the discussion to follow, we find it necessary to trade simplicity and readability for mathematical precision and completeness of detail. Our choices
are based on a desire to provide all details necessary for others to verify our work and duplicate our results.

Gathering the Data

The data required to solve equations (3) and (4) were obtained from the original papyri, located in the Church Historian’s Vault at the LDS Church History Library in Salt Lake City. The papyri are stored in a large presentation book, like a giant photo album. The book is designed so that one can look at the front side of the fragments and then, by turning the page, also look at the backside of the mounting paper. The fragments are encased in transparent Mylar and placed inside the presentation book’s transparent plastic sheaths. On visual inspection, the Hôr papyrus appears to be substantially thicker than modern paper and about twice as thick as the Tshenmîn papyrus. The papyrus quite visibly stands off the page wherever there is a clean edge. The thickness varies considerably, especially where the top (recto) layer has peeled away from the bottom (verso) layer.

For simplicity, we’ll adopt Edward Ashment’s naming convention for the Hôr fragments; i.e., pJS 1.1, 1.2, and 1.3, corresponding to Nibley’s I, XI, and X, respectively. We gathered our data by placing plastic transparencies directly over the papyri and tracing the edges of the fragments. The transparencies clung electrostatically to the plastic sheaths, keeping them stationary during the tracing procedure. The end results of this process are illustrated in Figures 2–4.

We scanned the tracings into Tag Image File (TIF) format and then boosted their contrast to facilitate edge detection. Then we digitized the high-contrast TIF files using software that assigned each pixel a value from 0 (white) to 255 (black). Our next step was to scan along each column of numbers, first from the top down to locate the upper edge of the papyrus and then from the bottom up to locate the lower edge of the papyrus. The end result of this procedure was a set of single-valued x–y functions for the top and bottom edges of each fragment. These functions were cross checked against the original tracings to ensure consistency and accuracy in the discretization. We denote the edge of each fragment as a distance function, Y(x), where x is the horizontal distance along the papyrus and Y is the vertical distance from the
Figure 2. Tracing of original pJS 1.1 superimposed on a scaled photograph of the papyrus fragment.
Figure 3. Tracing of original JJS 1.2 superimposed on a scaled photograph of the papyrus fragment.
Figure 4. Tracing of original pJS 1.3 superimposed on a scaled photograph of the papyrus fragment.
edge to a horizontal reference line through the middle of the papyrus. With \( Y(x) \) in hand, the winding lengths are determined by computing the autocorrelation function for the edge of each fragment.

Before proceeding with the correlation analysis, we briefly describe how the method works. Recall that damage inflicted on the wound-up Hôr scroll—presumably by Michael Chandler when he removed it from its embalming salve—imprinted a repeating pattern of lacunae in the extant papyrus. The distances between successive matching lacunae in the unrolled papyrus correspond to the lengths of the original windings; hence, the outer windings can be measured by shifting the edge function of each fragment, with respect to itself, until the lacunae match up. For example, imagine a section of papyrus that includes exactly two windings, with matching lacunae in each winding. To determine the average winding length for the section, we could simply shift the left-hand lacuna to the right until it matched the right-hand lacuna, or we could shift the right-hand lacuna to the left until it overlaid the left-hand lacuna; either way, the shifting distance would correspond to the average winding length for the section. This could be done by visual inspection if the shapes of the lacunae were very similar; however, a more precise and objective approach is to employ the autocorrelation function for automatic pattern detection. The autocorrelation function quantifies the strength of agreement between lacunae as the edge function is shifted with respect to itself. The shifting distance that produces the highest level of agreement between lacunae is the most probable average winding length for that section. The best agreement is represented as a peak (local maximum), as seen in the graphs on the following pages.

Once the winding lengths for a particular section are determined, we compute the winding numbers (not necessarily integers) to which they correspond. These data make it possible to determine the mean effective thickness (change in radius per winding) for each section. For pJS 1.3, the absolute winding numbers are unknown due to the gap between pJS 1.2 and 1.3. This is not a problem however, since we need only the relative winding numbers to calculate a mean effective thickness. Lastly, once the mean effective thickness has been determined for all extant sections, we
examine the consistency of the results and use equations (3) and (4) to find the length of the missing interior portion of the scroll.

**Correlation Analysis of pJS 1.1 and 1.2**

The correlation function is defined as

\[ C(\Delta x) = \frac{(YZ) - (Y)(Z)}{\sqrt{(Y^2) - (Y)^2} \sqrt{(Z^2) - (Z)^2}} \]  

(5)

where \( Z = Y(x - \Delta x) \) and the angle brackets denote spatial averages; i.e.,

\[ (Y) = \frac{1}{b-a} \int_a^b Y(x) \, dx \]  

(6)

where \( x = a \) and \( x = b \) are the left and right ends, respectively, of the region where \( Y(x - \Delta x) \) overlaps \( Y(x) \). (\( C \) is defined only in the overlapping region.) The correlation function quantifies the level of agreement between lacunae as the windings are shifted by \( \Delta x \).

For example, if \( Y(x) \) and \( Y(x - \Delta x) \) are in perfect agreement (which is obviously the case for \( \Delta x = 0 \)), then \( C \) will be 1. If there is no agreement whatsoever between \( Y(x) \) and \( Y(x - \Delta x) \) then \( C \) will be near 0. If \( Y(x) \) and \( Y(x - \Delta x) \) are exact opposites, then \( C \) will be 1.

The winding lengths thus correspond to the distances between local maxima of \( C(\Delta x) \).

In order to obtain a strong isolated peak in \( C(\Delta x) \), it is desirable that the region of overlap, which we’ll denote by \( \xi = b - a \), between \( Y(x) \) and \( Y(x - \Delta x) \), be a significant fraction of \( W_n \); however, \( \xi \) should not exceed \( W_n \) or else \( C(\Delta x) \) will contain contributions from regions unrelated to \( W_n \) (which would skew the results). With these principles and caveats in mind, we now proceed to compute \( C(\Delta x) \) and \( W_n \) for each fragment.

**Upper Edge**

The autocorrelation function for the upper edge of pJS 1.1 is shown in Figure 5. At \( \Delta x = 10.42 \) cm, it exhibits a strong local maximum of \( C = 0.91 \). Such a high correlation constitutes a virtual certainty that the peak corresponds to the local winding length. The similarity of the pJS 1.1 edge functions, \( Y(x) \) and \( Y(x - 10.42) \), in
Figure 5. Autocorrelation function for top edge of pJS 1.1.

Figure 6. Top edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x - 10.42)$ (dashed line), for pJS 1.1.
their region of overlap, is apparent in Figure 6. Here we have set \( x = 0 \) to correspond to the (top) junction between pJS 1.1 and 1.2.

The dashed line in the overlapping region, \( 10.42 \leq x \leq 18.72 \), passes through the chest and wrist of Anubis. (See Figure 2.) A few scholars have argued that the portions of the papyrus containing the head and knife were intact at the time Joseph Smith possessed it.\(^{28}\) Among the counterarguments that have been offered against this theory is that, if the edge of the papyrus were extended to include the head and knife, then the agreement between successive lacunae would be considerably degraded.\(^{29}\) The strong correlation shown here adds quantitative weight to this observation.

Now that we have the winding length for this fragment, we need to determine the exact winding number to which it corresponds. This depends on how much of the second winding is contained in the overlapping region. Let \( n_b \) be the winding number at \( x = b \) (the right end of the overlapping segment), then

\[
n = n_b + \frac{W_n + \xi}{2W_n}
\]

(Recall that \( n \) corresponds to the angular center of \( W_n \).) We have defined the right edge \( (x = 18.72 \text{ cm}) \) of pJS 1.1 as \( n = 0 \); hence, \( b = 18.72 \text{ cm}, n_b = 0, W_n = a = 10.42 \text{ cm} \) and \( \xi = 18.72 - 10.42 = 8.30 \text{ cm} \). Plugging these numbers into equation (7) yields \( n = 0.8983 \); i.e., \( W_{0.8983} = 10.42 \text{ cm} \). Since \( \xi \) is slightly less than \( W_n \), we have satisfied the overlap constraint discussed earlier.

The autocorrelation function for the upper edge of pJS 1.2 is displayed in Figure 7. Once again we observe a strong local maximum of \( C(10.34) = 0.90 \). The edge functions, \( Y(x) \) and \( Y(x - 10.34) \), for pJS 1.2 are given in Figure 8. For this fragment, \( \xi = 6.02 \text{ cm} \) and \( W_n = 10.34 \text{ cm} \).\(^{30}\) To get \( n_b \) (at \( x = 0 \)), we note that \( W_{0.8983} = 10.42 \text{ cm} \) corresponds to the average winding length for pJS 1.1; therefore, \( n_b = 18.72/10.42 = 1.797 \). Plugging these numbers into equation (7) yields \( n = 2.588 \); hence, \( W_{2.588} = 10.34 \text{ cm} \). Once again, \( \xi \) is slightly less than \( W_n \) and we have satisfied the overlap constraint. Figure 9 summarizes the results for the top edges of pJS 1.1 and 1.2. The upper-edge analysis of pJS 1.1 and 1.2 suggests that the mean effective thickness of the outer windings is \( T = (10.42 - 10.34)/[2\pi(2.588 - 0.8983)] = 0.0075 \text{ cm} \).
Figure 7. Autocorrelation function for top edge of pJS 1.2.

Figure 8. Top edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x - 10.34)$ (dashed line), for pJS 1.2.
Figure 9. Winding numbers, $n$, for upper edge of pJS 1.1 and 1.2. This outer section of the scroll comprises 3.379 windings. The horizontal double-headed arrows indicate the regions corresponding to the average winding lengths, $<W>$. 

Figure 10. Autocorrelation function for bottom edge of pJS 1.1.
Figure 11. Edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x - 10.49)$ (dashed line), for bottom edge of pJS 1.1.

Figure 12. Autocorrelation function for bottom edge of pJS 1.2.
Although the periodicity in the lacunae along the bottom edge is less obvious than along the top edge, it is nevertheless apparent that significant damage to the lower edge occurred while the scroll was still wound up. Much of this damage/decay undoubtedly occurred during the millennia of dormancy in the tomb. Additional damage to this end of the scroll may also have been caused by Chandler grasping/pulling/pushing the scroll from its wrappings. Whatever caused the damage, the distinctive pattern along the bottom edge of the scroll contains important information about the winding lengths.

Proceeding exactly as we did for the top edges, we calculate the autocorrelation function for the bottom edge of pJS 1.1 and find a winding length of 10.49 cm (Figure 10). The repeating pattern of lacunae along the bottom edge of pJS 1.1 is apparent in Figure 11. Here we have once again defined $x = 0$ as the (bottom) junction between pJS 1.1 and 1.2. (Note that this is not exactly the same horizontal location as for the top edge due to the angle of

Figure 13. Edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x-9.74)$ (dashed line), for bottom side of pJS 1.2.

**Lower Edge**

Although the periodicity in the lacunae along the bottom edge is less obvious than along the top edge, it is nevertheless apparent that significant damage to the lower edge occurred while the scroll was still wound up. Much of this damage/decay undoubtedly occurred during the millennia of dormancy in the tomb. Additional damage to this end of the scroll may also have been caused by Chandler grasping/pulling/pushing the scroll from its wrappings. Whatever caused the damage, the distinctive pattern along the bottom edge of the scroll contains important information about the winding lengths.

Proceeding exactly as we did for the top edges, we calculate the autocorrelation function for the bottom edge of pJS 1.1 and find a winding length of 10.49 cm (Figure 10). The repeating pattern of lacunae along the bottom edge of pJS 1.1 is apparent in Figure 11. Here we have once again defined $x = 0$ as the (bottom) junction between pJS 1.1 and 1.2. (Note that this is not exactly the same horizontal location as for the top edge due to the angle of
the cut between the fragments.) Similarly, we define $n$ and $W_n$ for the bottom edge independently of $n$ and $W_n$ for the top edge, with $n = 0$ again corresponding to the right end of the lower edge. From Figures 10 and 11, we have $b = 18.26$ cm, $n_b = 0$, $W_n = a = 10.49$ cm and $\xi = 18.26 - 10.49 = 7.77$ cm. Plugging into equation (7) yields $n = 0.8704$; thus, $W_{0.8704} = 10.49$ cm. Note how well this result agrees with the top winding. Since $\xi$ is slightly less than $W_n$, we have again satisfied the overlap constraint.

The autocorrelation function for the lower edge of pJS 1.2 is displayed in Figure 12. And the shifted lacunae for the bottom edge of pJS 1.2 are shown in Figure 13. For this section, $\xi = 7.01$ cm and $W_n = 9.74$ cm. To get $n_b$ (at $x = 0$ along the lower edge), we note that $W_{0.8704} = 10.49$ cm corresponds to the average winding length for the bottom of pJS 1.1; therefore, $n_b = 18.26/10.49 = 1.741$. Plugging these numbers into equation (7) yields $n = 2.601$; hence, $W_{2.601} = 9.74$ cm. Once again, $\xi$ is slightly less than $W_n$ and
Figure 15. Autocorrelation function for top edge of pJS 1.3A (5.35 ≤ x ≤ 20.13 cm).

Figure 16. Edge function, Y(x) (solid line), and shifted edge function, Y(x−8.48) (dashed line), for upper side of pJS 1.3A.
we have satisfied the overlap constraint. Figure 14 summarizes the results for the bottom edges of pJS 1.1 and 1.2.

The lower-edge analysis suggests that the mean effective thickness of the outer windings is $T = (10.49 - 9.74) / [2\pi(2.601 - 0.8704)] = 0.0690$ cm. Comparing the top and bottom winding lengths of pJS 1.1, we see that $W_{0.8983} = 10.42$ for the top edge agrees well with $W_{0.8704} = 10.49$ for the bottom edge (we expect lower winding numbers to be longer); however, for pJS 1.2, the top winding length of $W_{2.588} = 10.34$ cm appears anomalously large compared to the bottom winding length of $W_{2.601} = 9.74$ cm. Analysis of pJS 1.3 (in the next section) will help adjudicate this discrepancy.

**Analysis of pJS 1.3**

*Upper Edge*

Since the pJS 1.3 fragment contains over three windings, applying the correlation analysis to the entire segment would violate the maximum overlap constraint; therefore, it is necessary to divide the fragment into two sections (similar to pJS 1.1 and 1.2). From Figure 4, it is apparent that extra damage occurred to the top edge of this fragment, at both the left and right ends, after the scroll was unrolled. The missing piece at the right end became separated from the backing paper and was subsequently glued upside-down into pJS 2.6 (Nibley’s IV); however, its impression is still clearly apparent in the glue and hence we include this section in our analysis. Scattered fragments in the lacuna at the left end indicate that much of the papyrus that was once glued here has since flaked off of the backing paper. Since the original edge here is uncertain, we exclude this segment from our analysis to avoid corrupting the results.

For simplicity, we have again placed $x = 0$ at the left edge of the fragment. We denote the segment extending from $x = 5.35$ to 20.13 cm as section A (or pJS 1.3A), and the segment extending from $x = 13.98$ to 28.89 cm as section B (or pJS 1.3B). In order to make $\xi$ close to $W_n$, it is necessary to overlap the segments; by doing so, we improve the reliability of the correlation. (Think of $\xi$ as the number of statistical samples.) The overlapping of segments makes the bookkeeping for this section slightly more complicated; nonetheless, the procedure is essentially the same as be-
Figure 17. Autocorrelation function for top edge of pJS 1.3B (13.98 ≤ x ≤ 28.89 cm).

Figure 18. Edge function, Y(x) (solid line), and shifted edge function, Y(x−8.96) (dashed line), for upper side of pJS 1.3B.
fore. The correlation function for segment A is shown in Figure 15. Figure 16 displays the shifted edge function. We’ll denote the winding length for this section as \( W_A \). Since \( \xi = 6.30 \) is slightly less than \( W_A = 8.48 \) cm, our overlap constraint is satisfied. The winding number (\( n_b \)) at 20.13 cm is unknown at this point, so for now we’ll just refer to it as \( n_{20.13} \). From equation (7), the winding number for segment A is \( n_A = n_{20.13} + 0.8715 \).

The correlation function for section B is given in Figure 17. The shifted edge function is provided in Figure 18. For this segment, \( \xi = 5.95 \) and \( W_B = 8.96 \) cm. Since \( \xi \) is less than \( W_B \), the overlap constraint is satisfied. Once again, the winding number (\( n_b \)) at 28.89 cm is unknown; hence, we’ll simply refer to it as \( n_{28.89} \). From equation (7), the winding number for segment B is \( n_B = n_{28.89} + 0.8320 \).

Now that we have the winding lengths for segments A and B; i.e., \( W_A = 8.48 \) cm and \( W_B = 8.99 \) cm, we must determine the difference between their winding numbers; i.e., \( \Delta n_{AB} = n_A - n_B \); then we can use equation (4) to determine the effective thickness of pJS 1.3. Denoting the section from \( x = 20.13 \) to 28.89 cm as segment Q (the distance from the right end of segment A to the right end of pJS 1.3), the difference in winding numbers is \( \Delta n_{AB} = n_{20.13} + 0.8715 - n_{28.89} - 0.8320 = n_{20.13} - n_{28.89} + 0.0395 = \Delta n_Q + 0.0395 \), where \( \Delta n_Q = n_{20.13} - n_{28.89} \). The spiral length of segment Q is \( \Delta x_Q = 28.89 - 20.13 = 8.76 \) cm. Changes in spiral distance are related to changes in winding number according to

\[
W_n = -\frac{dx}{dn}
\]  

Hence, \( W_Q = -\frac{\Delta x_Q}{\Delta n_Q} \) or \( \Delta n_Q = 8.76/W_Q \), where \( W_Q \) is the mean winding length for segment Q. Although \( W_n \) is not a linear function of \( x \), a good first-order estimate of \( W_Q \) is obtained via the linear extrapolation

\[
\frac{W_Q - W_A}{W_B - W_A} = \frac{x_Q - x_A}{x_B - x_A}
\]  

where: \( x_A = (5.35 + 20.13)/2 = 12.74 \) cm, \( x_B = (13.98 + 28.89)/2 = 21.44 \) cm and \( x_Q = (20.13 + 28.89)/2 = 24.51 \) cm. Plugging the various lengths into equation (9) yields \( W_Q = 9.13 \) cm; hence \( \Delta n_Q = \)
0.959 and $\Delta n_{AB} = 0.999$. (Recall that $\Delta n_{AB}$ is the distance from the angular-center of winding A to the angular-center of winding B.) Equation (4) now gives the effective thickness of pJS 1.3 as $T = (W_B - W_A)/(2\pi\Delta n_{AB}) = 0.0765$ cm. Figure 19 summarizes the results for the top edge of pJS 1.3.

A brief summary of the top-edge results for pJS 1.3 may help clarify the arithmetic. First, recall that the number of windings in a segment is the length of the segment divided by the average winding length for the segment. Segment A is $20.13 - 5.35 = 14.78$ cm long and has a mean winding length of $W_A = 8.48$ cm; hence, the number of windings in this section is $14.78/8.48 = 1.743$. The segment begins at $n = \nu + 0.959$ and ends at $n = \nu + 0.959 + 1.743 = \nu + 2.702$. Segment B is $28.89 - 13.98 = 14.91$ cm long and has a
mean winding length of \( W_B = 8.96 \text{ cm} \); hence, the number of windings in this section is \( 14.91/8.96 = 1.664 \). The segment begins at \( n = v \) and ends at \( n = v + 1.664 \). In Figure 19, \( n_A \) would be just to the left of \( v + 1.664 \) and \( n_B \) would be just to the right of \( v + 0.959 \).

Lower Edge

For the bottom edge of pJS 1.3, we define our x-axis such that the edge extends from \( x = 0 \) to 29.97 cm. There is no reason to exclude any portion of the bottom edge from our analysis; thus, in order to meet the overlap constraint, we simply divide the entire segment in half. We define section A as the segment extending from \( x = 0 \) to 15 cm (14.99 due to the finite discretization) and sec-

\[ \text{Figure 20. Autocorrelation function for lower edge of pJS 1.3A (} 0 \leq x \leq 14.99 \text{ cm).} \]
Figure 21. Edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x - 8.06)$ (dashed line), for bottom side of pJS 1.3A.

Figure 22. Autocorrelation function for lower edge of pJS 1.3B ($15.01 \leq x \leq 29.97$ cm).
tion B as the segment extending from $x = 15$ (15.01 due to the discretization) to 29.97 cm. The correlation function for segment A is shown in Figure 20. And the shifted edge function is provided in Figure 21. For this segment, $\xi = 6.30 < W_A = 8.06$ cm, satisfying our overlap constraint. The winding number for segment $W_A$ is $n_A = n_{15} + 0.8908$.

The correlation function for section B is given in Figure 22. The local maximum of $C(8.99) = 0.27$ is significantly lower than the peaks for all of the other segments, which raises some concern. Figure 23 shows that the low correlation is caused by two prominent spikes (due to cracks extending into the papyrus) in the shifted edge function at $x = 27.7$ and 28.5 cm. Except for these spikes, the overall shape of the large dip, centered near $x = 28$ cm, is similar for both curves; hence, we can have confidence in the

![Figure 23. Edge function, $Y(x)$ (solid line), and shifted edge function, $Y(x - 8.99)$ (dashed line), for lower edge of pJS 1.3B.](image)
winding length despite the low correlation. For this segment, $\xi = 5.97 \, \text{cm}$, $W_B = 8.99 \, \text{cm}$ (satisfying the overlap constraint), and $n_B = n_{29.97} + 0.8320$ (recall that this winding number corresponds to the angular center of the $W_B$ winding). The winding number at $x = 15$ cm is $n_{15} = n_{29.97} + (29.97 - 15)/8.99 = n_{29.97} + 2.221$. Hence, $\Delta n_{AB} = n_{29.97} + 2.221 + 0.8908 - n_{29.97} - 0.8320 = 2.2798$ and $T = (8.99 - 8.06)/[2\pi(2.2798)] = 0.0649 \, \text{cm}$. Figure 24 summarizes the results for the bottom edge of pJS 1.3.

**The pJS 1.2 Top-Edge Outlier**

We now have the following four estimates of the effective thickness parameter:

$T = 0.0075 \, \text{cm}$ for the top edge of pJS 1.1 and 1.2

$T = 0.0690 \, \text{cm}$ for the bottom edge of pJS 1.1 and 1.2
Obviously, the first estimate is not consistent with the other three. In regards to the windings, we have consistency between the top and bottom winding lengths for pJS sections 1.1, 1.3A and 1.3B, but not for pJS 1.2. The bottom winding length for pJS 1.2 is consistent with the other bottom winding lengths; however, the top winding length for this section appears too long.

To resolve the discrepancy between the top and bottom winding lengths for pJS 1.2, we note prominent cracks in the papyrus beneath the lacunae in both pJS 1.1 and 1.2. The crack in pJS 1.1 passes just in front of the Horus crocodile and through the belly and nose of the Duamutef canopic jar, as shown in Figure 25. This crack wanders a bit but is located roughly 10.6 cm from the outside edge of the papyrus, which is very close to the expected length of the first winding \(W_{0.5}\). It may be that the scroll’s linen binding pressed the outer edge of the first wrapping into the wrappings beneath it, causing these cracks to appear, or the papyrus may have cracked when it was first unwound, as the outer edge was pried loose from the rest of the scroll.

In pJS 1.2, corresponding cracks appear beneath both of the major lacunae (Figure 3). These cracks appear to coincide with the ends of the second and third windings. The distance between them, as shown in Figure 26, is about 9.8 cm. A 9.8 cm winding length agrees well with the 9.74 cm average winding length that we obtained from our lower-edge correlation for this fragment, but not with the 10.34 cm winding length that we obtained from the upper-edge analysis.

The anomalous upper-edge winding length for pJS 1.2 appears to be the result of damage inflicted on the upper-right portion of the fragment after the scroll was unrolled. Although the characters at the beginnings of lines 1 and 2 of the instructions column on pJS 1.2 are now missing, enough of these characters were extant in 1835 that Smith’s scribes could copy them into the Book of Abraham translation manuscripts. Additionally, the first two characters on line 1 were copied into the Egyptian Alphabet
Figure 25: Close-up of crack at the end of the first winding.
Figure 26. Close-up of cracks beneath the major lacunae in pJS I.2.
Figure 27. Characters that were extant on pJS 1.2 in 1835 (red boxes) are shown restored, based on other instances of the same character groupings (blue boxes) on the same fragment. For this illustration in color, see www.dialoguejournal.com.
and Grammar manuscripts. Regarding line 1, Edward Ashment writes:

From the beginning of column 1 line 1 of pJS 1.2, Smith transcribed the [first two] (now badly damaged) hieratic characters . . . A parallel Breathing Permit reveals that the [first two] characters . . . originally were part of a three-character group . . . Unfortunately, the third sign . . . already was missing in a lacuna when Smith worked on his “Egyptian Alphabet” although, near the end of line three of the papyrus, the same sign group appears in its entirety.31

With respect to line 2, Klaus Baer observes:

The missing signs occur again on the same photograph in ii, 3, to the left of the break, starting with the group after the short horizontal dash and continuing to the end of the preserved part of the line. Joseph Smith [in the Book of Abraham translation manuscripts] drew four groups, of which the first ("Behold Potipher’s hill . . .") has the expected shape and is still visible in traces at the beginning of the line, while the remaining three (including the one corresponding to Abraham 1:26) are clearly proposed restorations that bear no resemblance to the signs that certainly were on the papyrus before it was damaged; note also the difference in general appearance or style. Our conclusion is essentially the same as before: The papyrus was slightly better preserved at the beginning of the line but otherwise broke off at the same point it does now.32

Figure 27 shows these characters restored, roughly as they would have appeared in 1835. The additional damage is not surprising, since this was likely the most frequently handled area of the papyrus. The second character in line 1 (a hieratic “w”) is translated as the name of Abraham in the 1835 Egyptian Alphabet and Grammar and Book of Abraham manuscripts.33 Joseph was fond of pointing to this character when visitors came to view the papyri. One visitor to Nauvoo in 1840 reported that the Prophet pointed to a particular character and announced, “There, . . . that is the signature of the patriarch Abraham.”34 Others similarly reported being shown “the handwriting of Abraham.”35 It could be that Joseph’s frequent handling of this portion of the papyrus caused some of the damage to this section.36

Whatever the cause, it appears that the extra damage to pJS 1.2 has shifted the rightmost edge of the lacuna in the instructions column over to the right. This means that the lacuna in the
next column over has to be shifted by an additional amount in order for the lacunae to match up. The increased shifting distance, needed to obtain a peak in the correlation function, results in an anomalously high winding length, which, in turn, causes T to be underestimated. We thus reject the $T = 0.0075$ cm estimate and take the average of the remaining three measurements to obtain $T = 0.0701$ cm. This effective thickness is in good agreement with the value of $\sim 0.8$ mm reported by Hartmut Stegemann for most of the Dead Sea papyrus scrolls.\textsuperscript{37}

**Lost Papyrus**

Plugging our effective thickness estimate into equation (3) returns the maximum possible length of the scroll, interior to winding $n_A$, on the bottom edge of $\text{pJ}S$ 1.3:
The distance from $n_A$ to the left edge of the innermost extant fragment (a piece glued upside-down into pJS 2.6) is $\approx 9.9$ cm. (See Figure 28.) Therefore, no more than 56 cm of papyrus can be missing from the scroll’s interior.

Shortly after the papyri were recovered by the LDS Church, Klaus Baer estimated the original length of the Hôr scroll to have been 150 to 155 cm. He arrived at this estimate by comparing the text to other copies of the Document of Breathing, particularly Papyrus Louvre 3284. Baer allowed 21 cm for column iv, of which 14 cm (including the misplaced piece) are extant. He estimated 35 cm for columns v and vi, 16 cm for Facsimile 3, “and a small amount for margin around the latter.”

Assuming half a centimeter margin on both sides of Facsimile 3, Baer’s estimate for the length of papyrus missing from the scroll’s interior, starting from the left edge of the innermost extant fragment, is $21 - 14 + 0.5 + 16 + 0.5 = 59$ cm. This estimate agrees remarkably well with the 56 cm obtained from our winding analysis. The 3 cm difference between Baer’s text-based estimate and our geometric estimate is within Baer’s 5 cm tolerance for the scroll’s overall length. Our results thus corroborate Baer’s estimate of $\approx 150$ cm for the total original length of the scroll of Hôr.

The lack of sufficient space for a Book of Abraham source text on the Hôr scroll raises the question of whether such a text might have been on another scroll or fragment in the original collection. This hypothesis appears unlikely, since the canonized Book of Abraham specifically places the introductory vignette of the Hôr Document of Breathing at its “commencement” (Abr. 1:12, 14). Moreover, the most reliable nineteenth-century eyewitnesses spoke of only two intact scrolls in Joseph Smith’s collection: the scrolls of Hôr and Tshenmîn. It is clear from the witnesses’ descriptions of the scrolls that the former was believed to contain the Book of Abraham, and the latter the Book of Joseph. Several eyewitnesses were also shown mounted fragments that were identified as Abrahamic writings. These were evidently the extant fragments from the fragile outer end of the Hôr scroll. Charlotte Haven’s description of “the writing of Abraham and Isaac” as “a long roll of manu-
script” suggests that the Hôr document was the longer of the two scrolls in Joseph’s possession. However, it should also be recognized that, with no congruous reference available to form an impression, “long” to Charlotte likely meant anything longer than the paper on which she wrote to her mother.42

In recognition of the unlikelihood that there ever was a Book of Abraham source text on the inner section of the Hôr scroll, several alternative theories have been put forth to the effect that: (1) the Document of Breathing served as a mnemonic device for the Book of Abraham, (2) the Breathing text served as a catalyst (rather than source text) for the Book of Abraham, (3) the Document of Breathing is a corrupted version of the Book of Abraham, which Smith restored to its pristine state, or (4) the Book of Abraham is simply an imaginative mistranslation of the hieratic script.43 The ultimate success of any existing or future theory will depend on its ability to account for all of the evidence, including the fact that there was simply no room on the papyrus for anything besides the Breathing text.

Irrespective of Joseph’s method of translation, it is clear that he sensed in the Hôr scroll a richness of symbolic and religious potential that contemporary scholars could not see. To the experts who viewed Chandler’s collection in New York and Philadelphia, the Hôr scroll was a cryptic relic of a dead religion from a dusty tomb. Joseph, however, breathed fresh meaning into the crumbling little scroll, giving it new life as powerful scripture for the latter days. Perhaps the Egyptian vision of the afterlife, described in Hôr’s Document of Breathing, is not so far-fetched after all.

Notes


13. See also Grant S. Heward and Jerald Tanner, “The Source of the


19. Oliver Cowdery, Letter to Frye, December 22, 1835, expected the completed translation to fill “large volumes.” This expectation was apparently influenced by a belief that Egyptian script was an extremely compact (“comprehensive”) form of writing in which a single character might be translated by dozens or even hundreds of English words. This is the understanding outlined in a nineteenth-century Egyptian Alphabet and Grammar (EAG) that seems to have been produced under Smith’s direction. Advocates of the “missing papyrus theory” however, have contested both this interpretation of the EAG and Smith’s role in its production. Missing papyrus theorists assume that the Book of Abraham is a more or less Egyptologically correct translation of a now-lost hieratic or hieroglyphic text, and thus (by implication) that the proportion of Egyptian to English text would not have been exceptional. See Hugh W. Nibley, “The Meaning of the Kirtland Egyptian Papers,” *BYU Studies* 11, no. 4 (Summer 1971): 350–99; Christopher C. Smith, “The Dependence of Abraham 1:1–3 on the Egyptian Alphabet and Grammar,” *John Whitmer Historical Association Journal* 29 (2009): 43–47.


21. He adds, “The actual value permitted for all intents and purposes lies higher. I might have recommended over 3 cm.” See Hoffmann, “Die Länge Des P. Spiegelberg,” 150. Quotations from Hoffmann are Christopher Smith’s translation from the German.
22. The effective thickness will be larger than the physical thickness of the papyrus, since air, debris, embalming salve, inhomogeneities, etc. may all intervene between windings and increase the radial distance between them. Furthermore, since the moisture content of papyrus drops over time and the parenchyma cell matrix decays from ultraviolet radiation and other forces of erosion, the extant papyrus fragments may be substantially thinner today than they were two millennia ago. The effective thickness more properly characterizes the radial increment between windings for the ancient scroll in its wound-up state.

23. We are grateful to Glenn N. Rowe for his assistance in viewing the papyri on November 23, 2009.


25. We were not allowed to photograph the papyri during the visit, so here we have superimposed the tracings onto scaled images of the papyri. The slight mismatches in some areas are due to photographic distortion. We are grateful to Brent L. Metcalfe for providing these photographs, which we believe to be the originals on which the Improvement Era sepia images were based. The uncropped photographs of pJS 1.2 and 1.3 contain rulers with distinctive tick marks precisely matching the tick marks on the rulers in the Improvement Era images. However, the rulers in the uncropped photographs are in different locations than those shown in the sepias, suggesting that they were repositioned sometime later in the production process. The uncropped photograph of pJS 1.1, unlike the Improvement Era sepia, does not contain a ruler. The Improvement Era prints appear to be photographs of photographs. Measurements taken from the rulers in the Improvement Era images do not precisely match the originals.

26. It is possible that the horizontal reference lines are off by a degree or two with respect to the direction of roll. Errors in winding length due to visual misalignment go like $\sin(\phi)$ (a very small number for small angles), where $\phi$ is the angle between the reference line and the roll direction. These errors strongly cancel when the windings are subtracted; hence, they are expected to lie well beneath the specified accuracy of our measurements.

27. This is guaranteed by the right-left symmetry of the procedure, provided that all points are weighted equally in determining the agreement.


30. We suspect that this length is anomalous for reasons discussed in the “Outlier” section of this paper.

31. Ashment, “Joseph Smith’s Identification of Abraham,” 123, 125. See also Nibley, “Meaning of the Kirtland Egyptian Papers,” 384. Baer drew a contrary conclusion in “The Breathing Permit of Hôr,” 129, but only because he did not have access to a crucial portion of the manuscript evidence.


36. The fragments were reportedly preserved under glass on the above-cited occasions, so if Joseph’s handling of pJS 1.2 caused the damage in this area, it was probably at some other time, either prior to framing or after removal from the frame.


39. We are happy to make our data available to anyone interested in verifying our results.


42. Since only the now-lost inner portion of the Hôr document was still intact as a scroll at the time, what Charlotte saw could not have been longer than about two feet. Her use of the term “manuscript” to describe the papyrus suggests that she evaluated the scroll’s length relative